

Recent theoretical improvement of hadronic $B_{(s)}$ decays

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In this mini-review, we show that a lot of theoretical efforts have been made for the theoretical study of two body hadronic $B_{(s)}$ and B_c decays. In addition to many next-to-leading order or even next-to-next-to leading order α_s corrections made, we also study many of the previously unknown next-to-leading order power corrections. While the former corrections are theoretically solid, the latter corrections are phenomenologically more important. In the QCD factorization approach based on collinear factorization, there is difficulty to deal with the power correction diagrams due to the endpoint singularity. Thus many of these analysis use phenomenological method. In the perturbative QCD approach based on k_T factorization, the endpoint singularity is killed by including the quark transverse momentum. Therefore we can calculate the annihilation type diagrams quantitatively, which give the right sign for the direct CP asymmetry parameters. More and more $B_{(s)}$ decays channels, especially the pure annihilation type decays have been measured by the recent experiments to confirm our theoretical predictions. More channels are predicted for future experiments, such as the charmless and charmed B_s and B_c hadronic decays and the decays involving a scalar, axial vector, even a tensor meson in the final states.

I. INTRODUCTION

In the LHC era of particle physics, heavy flavor physics is still one of the hot topics in particle physics. Most of the standard model free parameters belong to the flavor part. One of the four experiments, the LHCb experiment focus mainly on the B physics, although other two experiments-ATLAS and CMS also providing us rich data of B physics. In fact, while the two B factories continue to analyze data, two new super B factories are preparing for 100 more luminosity. With the big achievements in experiments, the theoretical improvement of B physics has been a little bit slow down.

Although many of the next-to-leading order or even next-to-next-to-leading order α_s corrections have been done in the perturbative QCD approach [1], the QCD factorization approach and the soft-collinear-effective theory study [2], the phenomenologically more important power corrections have not been systematically studied in these approaches. It has been demonstrated that in many non-leptonic B decays, the previously missed power correction, such as annihilation type diagrams are very important in the direct CP asymmetry and polarization study of vector meson final states [3]. As pointed out, these power corrections are not calculable in the QCD factorization approach with endpoint singularity [4]. In fact, the well defined perturbative QCD approach with k_T factorization [5] can calculate all of these kinds of diagrams without ambiguity.

In two body hadronic $B_{(s)}$ decays, the light final state mesons and their constituent quarks inside are collinear objects at the rest frame of $B_{(s)}$ meson. The light spectator quark in the $B_{(s)}$ meson is rather soft. Therefore a hard gluon is needed to transform it into a collinear object. The dominant contribution is perturbative. However, the calculation is sometimes divergent at the endpoint of the meson distribution amplitudes. To deal with this unphysical singularity, one leaves the form factor diagrams to non-perturbative contributions in QCD factorization approach and soft-collinear effective theory (SCET) [6]. For the annihilation type diagrams, the QCD factorization approach just parameterizes it as free parameters; while SCET argues it as small power corrections. In fact, the quark carries very little longitudinal momentum at the endpoint, therefore the transverse momentum of quark is no longer negligible. In the perturbative QCD approach, we keep the transverse momentum of quark, which acts as a natural regulator of the endpoint divergence. Including another momentum scale (transverse momentum) in QCD will produce large double logarithms in the perturbative calculations, so that we have to use the renormalization group equation to do the resummation. A Sudakov factor is produced, which suppresses the endpoint contributions

to keep the perturbative calculation healthy.

In this paper, we will show that many of the leading order and also next-to-leading order perturbative QCD calculations of $B_{(s)}$ and B_c decays have been tested in the experiments and more and more channels are predicted for future experiments. All of the charmless or charmed meson decays of B_s meson have been studied some years ago in the perturbative QCD approach [7, 8] and the QCD factorization approach [3, 4]. Some of the charmless B_s decays are also discussed in the soft-collinear effective theory [9, 10]. Recently, a scalar meson, axial vector meson, or tensor meson involved in the final states are also studied. Since the LHC experiment will produce numerous B_c mesons, the intensive study of B_c meson hadronic decays are also studied, which include the charmless final states, one or two D mesons in the final states and even final states involving scalar or tensor mesons.

II. FACTORIZATION METHOD IN QCD FOR HADRONIC $B_{(s)}$ DECAYS

All the hadronic $B_{(s)}$ decays are weak decays. In the quark level, the weak transitions are summarized as effective four quark operators with QCD corrections. The weak effective Hamiltonian \mathcal{H}_{eff} can be written as [11]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q=u,c} V_{qb}^* V_{qX} [C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu)] - V_{tb}^* V_{tX} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\}, \quad (1)$$

with $V_{qb(X)}$ and $V_{tb(X)}$ ($X = d, s$) the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The current-current (tree) local four-quark operators are:

$$O_1^q = (\bar{b}_\alpha q_\beta)_{V-A} (\bar{q}_\beta X_\alpha)_{V-A}, \quad O_2^q = (\bar{b}_\alpha q_\alpha)_{V-A} (\bar{q}_\beta X_\beta)_{V-A}; \quad (2)$$

the QCD penguin operators are

$$O_3 = (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad O_4 = (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (3)$$

$$O_5 = (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad O_6 = (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (4)$$

and the electro-weak penguin operators are

$$O_7 = \frac{3}{2} (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad O_8 = \frac{3}{2} (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (5)$$

$$O_9 = \frac{3}{2} (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad O_{10} = \frac{3}{2} (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (6)$$

where α and β are the color indices and q' are the active quarks at the scale m_b , i. e. $q' = (u, d, s, c, b)$. The left-handed and right-handed currents are defined as $(\bar{b}_\alpha q_\beta)_{V-A} = \bar{b}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta$ and $(\bar{q}'_\beta q'_\alpha)_{V+A} = \bar{q}'_\beta \gamma_\mu (1 + \gamma_5) q'_\alpha$, respectively. The Wilson coefficients C_i^q s are calculated by the renormalization group equations to include the next-to-leading order QCD corrections.

When doing the hadronic matrix elements calculations, we need to deal with scales around $\sqrt{\Lambda_{QCD} m_b}$, which is usually called factorization scale. Scales below this scale are categorized as non-perturbative physics, which is described by form factors or meson wave functions. In the generalized factorization approach [12], the QCD corrected Wilson coefficients are scale dependent while the matrix elements described by the form factors are scale independent physical quantities. It is later improved by the QCD factorization approach [4] to remove the scale dependence in the decay amplitudes by including the vertex corrections together with the hard scattering diagrams shown at Fig.1(c) and (d). In this collinear factorization based theory, however, there is endpoint singularity at the higher twist calculation and the annihilation type diagrams shown in Fig2. Those annihilation type diagrams are later proved to be important [3]. Similar to the QCD factorization approach, the soft-collinear effective theory [6] also leave part of the soft contribution in the form factor diagrams shown in Fig.1(a) and (b) as non-perturbative

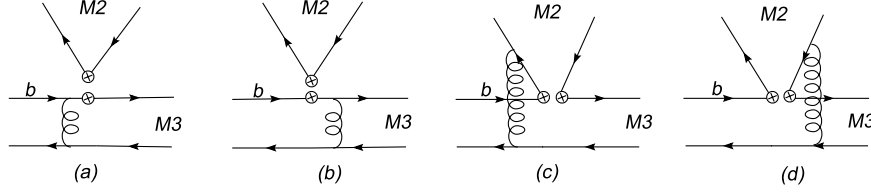


FIG. 1: The leading order emission tip Feynman diagrams

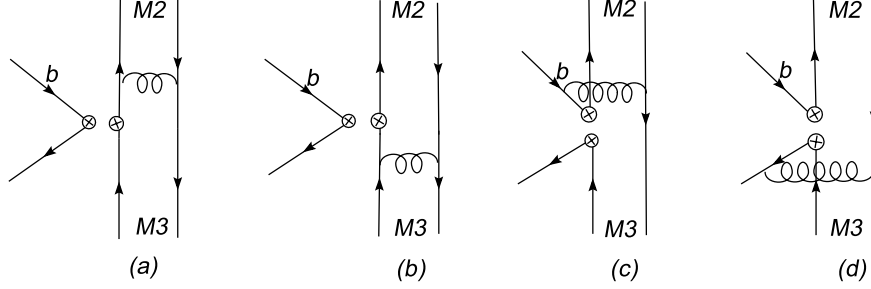


FIG. 2: The leading order annihilation type Feynman diagrams

contribution. This makes the soft-collinear effective theory less predictive, since it requires more free parameters to be determined by experiments [9, 10].

In the perturbative QCD approach, we keep the transverse momentum of quarks to kill the endpoint divergence. Therefore, we have one more scale, i.e. the quark transverse momentum than the QCD factorization approach and SCET. The factorization formula in pQCD approach is then

$$C(t) \otimes H(x, t) \otimes \Phi(x) \otimes \exp[-s(P, b) - 2 \int_{1/b}^t \frac{d\mu}{\mu} \gamma_q(\alpha_s(\mu))], \quad (7)$$

where $C(t)$ are the corresponding Wilson coefficients of four quark operators, which include the dynamics of physics larger than m_b scale. The Sudakov evolution $\exp[-s(P, b)]$ [13] are from the resummation of double logarithms $\ln^2(Pb)$, with P denoting the dominant light-cone component of meson momentum. $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension in axial gauge. All non-perturbative components are organized in the form of hadron wave functions $\Phi(x)$, which may be extracted from experimental data or other non-perturbative method, such as QCD sum rules. Since non-perturbative dynamics has been factored out, one can evaluate all possible Feynman diagrams for the six-quark amplitude straightforwardly, which include both factorizable and non-factorizable emission contributions shown in Fig.1. Factorizable and non-factorizable annihilation type diagrams shown in Fig.2 are also calculable without endpoint singularity.

A. Wave Functions of the B_s Meson

In order to calculate the analytic formulas of the decay amplitudes, we need the light cone wave functions decomposed in terms of the spin structure. In general, the light cone wave functions are decomposed into 16 independent components, $1_{\alpha\beta}$, $\gamma_{\alpha\beta}^\mu$, $\sigma_{\alpha\beta}^{\mu\nu}$, $(\gamma^\mu \gamma_5)_{\alpha\beta}$, $\gamma_{5\alpha\beta}$. If the considered meson is the heavy B or B_s meson, the $B_{(s)}$ meson light-cone matrix element can be decomposed as [14, 15]

$$\begin{aligned} & \int d^4z e^{ik_1 \cdot z} \langle 0 | \bar{b}_\alpha(0) s_\beta(z) | B_s(P_{B_s}) \rangle \\ &= \frac{i}{\sqrt{6}} \left\{ (\not{P}_{B_s} + M_{B_s}) \gamma_5 \left[\phi_{B_s}(k_1) - \frac{\not{n} - \not{k}}{\sqrt{2}} \bar{\phi}_{B_s}(k_1) \right] \right\}_{\beta\alpha}. \end{aligned} \quad (8)$$

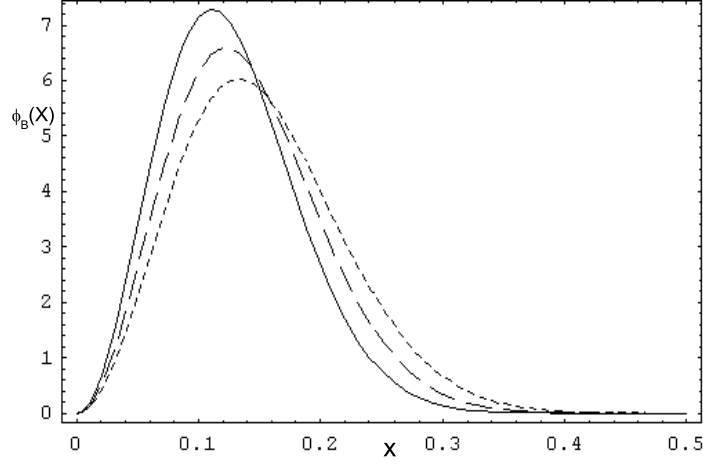


FIG. 3: B_s meson distribution amplitudes. The solid-, dashed-, and tiny-dashed- lines correspond to $\omega_B = 0.45$ GeV, 0.5 GeV, and 0.55 GeV, respectively.

From the above equation, one can see that there are two Lorentz structures in the B_s meson distribution amplitudes, that obey the following normalization conditions

$$\int \frac{d^4 k_1}{(2\pi)^4} \phi_{B_s}(k_1) = \frac{f_{B_s}}{2\sqrt{6}}, \quad \int \frac{d^4 k_1}{(2\pi)^4} \bar{\phi}_{B_s}(k_1) = 0. \quad (9)$$

In general, one should consider these two Lorentz structures in the calculations of B_s meson decays. Since the contribution of $\bar{\phi}_{B_s}$ is numerically small [16], its contribution can be neglected. With this approximation, we only keep the first term in the square bracket from the full Lorentz structure in Eq. (8)

$$\Phi_{B_s} = \frac{i}{\sqrt{6}} (\not{P}_{B_s} + M_{B_s}) \gamma_5 \phi_{B_s}(k_1). \quad (10)$$

Usually the hard part is always independent of one of the k_1^+ and/or k_1^- . The B_s meson wave function is then a function of the variables k_1^- (or k_1^+) and k_1^\perp only,

$$\phi_{B_s}(k_1^-, k_1^\perp) = \int \frac{dk_1^+}{2\pi} \phi_{B_s}(k_1^+, k_1^-, k_1^\perp). \quad (11)$$

The B_s meson's wave function in the b -space can be expressed by

$$\Phi_{B_s}(x, b) = \frac{i}{\sqrt{6}} [\not{P}_{B_s} \gamma_5 + M_{B_s} \gamma_5] \phi_{B_s}(x, b), \quad (12)$$

where b is the conjugate space coordinate of the transverse momentum k^\perp .

In this study, we use the model function similar to that of the B meson which is

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp \left[-\frac{M_{B_s}^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \quad (13)$$

with N_{B_s} the normalization factor. In recent years, a lot of studies have been performed for the B_d^0 and B^\pm decays in the pQCD approach [5]. The parameter $\omega_b = 0.40$ GeV has been fixed using the rich experimental data on the B_d^0 and B^\pm mesons. In the SU(3) limit, this parameter should be the same in B_s decays. However, facing the high precision experimental data, one has to consider the small SU(3) breaking effect, i.e. the s quark momentum fraction should be a little larger than that of the u or d quark in the lighter B mesons. The shape of the distribution amplitude is shown in Fig.3 for $\omega_B = 0.45$ GeV, 0.5 GeV, and 0.55 GeV. It is easy to see that the larger ω_b gives a larger momentum fraction to the s quark. We use $\omega_b = 0.50 \pm 0.05$ GeV in this paper for the B_s decays [7].

B. B_c meson wave function

For B_c meson, we only consider the contribution from the first Lorentz structure, like B_q ($q = u, d, s$) meson,

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}(\not{P} + m_{B_c})\gamma_5\phi_{B_c}(x, b). \quad (14)$$

Consisting of two heavy quarks (b,c), the B_c meson is usually treated as a heavy quarkonium system. In the non-relativistic limit, we adopt the model for the distribution amplitude as[29]:

$$\phi_{B_c}(x, b) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - m_c/m_{B_c})\exp\left[-\frac{1}{2}w^2b^2\right], \quad (15)$$

in which $\exp\left[-\frac{1}{2}w^2b^2\right]$ represents the k_T dependence. f_{B_c} and $N_c = 3$ are the decay constant of B_c meson and the color number respectively.

For the final state meson wave functions, such as pion and Kaon, we refer the readers to the papers dealing with various decay channels [5, 18]. For the heavy D meson in the final states, we utilize the heavy quark symmetry to simplify the Lorentz structure [8].

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations, one needs wave function parameters and form factors in the QCD factorization approach as input to give numerical results. To deal with the annihilation type diagrams, one needs more parameters preferably fitted by experiments. In the soft-collinear effective theory, one needs to fix even more parameters by fitting experimental data [9, 10], since there are more contributions such as charming penguins without theoretical predictions. For the perturbative QCD approach discussed more thoroughly in this paper, one needs only wave function parameters. Since some parameters change time to time, one needs to consult the original paper for input parameters for each predictions. We do not list them here one by one.

A. Charmless hadronic two body decays of B_s meson

In the SU(3) symmetry limit, the B_s is very similar to the B^0 meson, which is also called the U-spin symmetry. However, the precision of current experimental measurement has already reached the size of SU(3) breaking effect in theoretical calculations. In ref.[7], we performed a systematic study of all the charmless $B_s \rightarrow PP, PV, VP$ and VV decays (here P and V stand for the light pseudo-scalar and vector mesons, respectively). After our predictions, some of the channels are measured by the experimental data [21], which are shown in table I. The theoretical errors for these entries correspond to the uncertainties in the input hadronic quantities, from the scale-dependence, and the CKM matrix elements, respectively. For comparison, we also cite the theoretical estimates of the branching ratios in two kinds of QCD factorization framework: QCDF I [4] & QCDF II [3], and in SCET [9]. Among them, the $B_s \rightarrow \pi^+\pi^-$ is the first channel of annihilation type B_s decays. The measured value is well consistent with our pQCD predictions. In the soft-collinear effective theory, the annihilation type diagrams are argued to be small and neglected, thus there is no prediction for this pure annihilation type decays. Up to now, there is also no SCET calculations for the two vector final state decays. As mentioned in the introduction, the annihilation type contributions are difficult to predict in QCD factorization approach, since there is endpoint singularity in these diagrams. A new QCD factorization analysis has been performed after the experimental measurement of $B_s \rightarrow \pi^+\pi^-$, which confirms that a large annihilation contribution is needed [17]. This annihilation B_s decay is a further confirmation of other annihilation type B^0 decays, such as $B^0 \rightarrow D_s^- K^+$, which is also well consistent between pQCD theory and experimental measurements [18].

In addition to the branching ratios, which have large theoretical and experimental uncertainties, there is also a first time direct CP asymmetry measurement in the $B_s \rightarrow K^-\pi^+$ decay [19]

$$A_{CP} = 27 \pm 8 \pm 2\%. \quad (16)$$

TABLE I: Some of the CP -averaged branching ratios ($\times 10^{-6}$) of $B_s \rightarrow PP$ and VV decays obtained in the pQCD approach (This work); the errors for these entries correspond to the uncertainties in the input hadronic quantities, from the scale-dependence, and the CKM matrix elements, respectively. We have also listed the updated experimental measurements [21]. For comparison, we also cite the theoretical estimates of the branching ratios in two kinds of QCD factorization framework: QCDF I [4] & QCDF II [3], and in SCET [9].

Modes	Class	QCDF I	QCDF II	SCET	This work	Exp.
$\overline{B}_s^0 \rightarrow K^+ \pi^-$	T	$10.2^{+4.5+3.8+0.7+0.8}_{-3.9-3.2-1.2-0.7}$	$5.3^{+0.4+0.4}_{-0.8-0.5}$	$4.9 \pm 1.2 \pm 1.3 \pm 0.3$	$7.6^{+3.2+0.7+0.5}_{-2.3-0.7-0.5}$	5.4 ± 0.6
$\overline{B}_s^0 \rightarrow K^+ K^-$	P	$22.7^{+3.5+12.7+2.0+24.1}_{-3.2-8.4-2.0-9.1}$	$25.2^{+12.7+12.5}_{-7.2-9.1}$	$18.2 \pm 6.7 \pm 1.1 \pm 0.5$	$13.6^{+4.2+7.5+0.7}_{-3.2-4.1-0.2}$	24.5 ± 1.8
$\overline{B}_s^0 \rightarrow \pi^+ \pi^-$	ann	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	$0.26^{+0.00+0.10}_{-0.00-0.09}$	—	$0.57^{+0.16+0.09+0.01}_{-0.13-0.10-0.00}$	0.73 ± 0.14
$\overline{B}_s \rightarrow \phi \phi$	P	$21.8^{+1.1+30.4}_{-1.1-17.0}$	$16.7^{+2.6+11.3}_{-2.1-8.8}$	-	$35.3^{+8.3+16.7+0.0}_{-6.9-10.2-0.0}$	19 ± 5
$\overline{B}_s \rightarrow K^{*0} \overline{K}^{*0}$	P	$9.1^{+0.5+11.3}_{-0.4-6.8}$	$6.6^{+1.1+1.9}_{-1.4-1.7}$	-	$7.8^{+1.9+3.8+0.0}_{-1.5-2.2-0.0}$	28.1 ± 7.2

Our calculations in ref.[7] give the direct CP asymmetry as

$$A_{CP} = 24.1^{+3.9+3.3+2.3\%}_{-3.6-3.0-1.2\%}, \quad (17)$$

while the QCD factorization approach gives a result with a minus sign if not fixing the annihilation diagram contribution [4]

$$A_{CP} = -6.7^{+2.1+3.1+0.2+15.5\%}_{-2.2-2.9-0.4-15.2\%}. \quad (18)$$

It is easy to see that the pQCD results agree with the experimental measurement quite well, which means that only the pQCD approach gives the right sign of strong phase, since the direct CP asymmetry is proportional to the sine of strong phase difference. This is a further example of the right prediction of direct CP asymmetry in pQCD after the $B^0 \rightarrow K^+ \pi^-$ and $B^0 \rightarrow \pi^+ \pi^-$ decays [20]. The last large theoretical uncertainty in the QCD factorization result is from the annihilation type diagram contribution, that is only a non calculable parameter. The later QCD factorization with fixed large contribution from annihilation diagrams gives similar results with the pQCD predictions [3]: $A_{CP} = 20.7^{+5.0+3.9\%}_{-3.0-8.8\%}$.

B. Hadronic two body B decays with one tensor meson in the final states

Recently, several experimental measurements about charmless B decay modes involving a light tensor meson (T) in the final states have been observed [22]. These decays have been studied in the naive factorization approach [23] and relativistic quark model [24], with which it can be easily shown that $\langle 0 | j^\mu | T \rangle = 0$, where j^μ is the $(V \pm A)$ or $(S \pm P)$ current [25]. The factorizable amplitude with a tensor meson emitted vanishes. So these decays are prohibited in the naive factorization approach. The branching ratios predicted in the naive factorization approach are too small compared with the experimental results, which implies the importance of nonfactorizable and annihilation type contributions. The recent QCD factorization (QCDF) approach analysis [25] proved this. It is worth of mentioning that the perturbative QCD (PQCD) approach [5] is almost the only method to calculate these kinds of diagrams, without fitting the experiments.

The numerical results of $B \rightarrow PT$ decays with $\Delta S = 1$, together with Isgur-Scora-Grinstein-Wise II (ISGW2) model [23] and QCDF results [25] are shown in table II. The experimental data are from Ref.[22]. The results of $B \rightarrow PT$ decays with $\Delta S = 0$ and also the CP asymmetry parameters for all of these decays can be found in ref.[26]. Among the considered $B \rightarrow PT$ decays, the PQCD predictions for the CP-averaged branching ratios vary in the range of 10^{-5} to 10^{-8} . From the numerical results, we can see that the predicted branching ratios of penguin-dominated $B \rightarrow PT$ decays in PQCD are larger than those of naive factorization [23] by one or two orders of magnitude, but are close to the QCDF predictions [25]. For illustration, we classify these decays by their dominant

TABLE II: The PQCD predictions of CP-averaged branching ratios (in units of 10^{-6}) for $B \rightarrow PT$ decays with $\Delta S = 1$, together with Isgur-Scora-Grinstein-Wise II (ISGW2) model [23] and QCDF results [25]. The experimental data are from Ref.[22].

Decay Modes	class	This Work	ISGW2 [24]	QCDF [4]	Expt.
$B^+ \rightarrow K_2^{*0} \pi^+$	PA	$0.9^{+0.2+0.2+0.3}_{-0.2-0.2-0.2}$...	$3.1^{+8.3}_{-3.1}$	$5.6^{+2.2}_{-1.4}$
$B^+ \rightarrow K_2^{*+} \pi^0$	PA	$0.4^{+0.1+0.1+0.1}_{-0.0-0.1-0.1}$	0.090	$2.2^{+4.7}_{-1.9}$...
$B^+ \rightarrow a_2^0 K^+$	T,PA	$2.1^{+0.7+0.6+0.6}_{-0.6-0.5-0.5}$	0.31	$4.9^{+8.4}_{-4.2}$	< 45
$B^+ \rightarrow a_2^+ K^0$	PA	$3.1^{+0.9+0.9+1.1}_{-0.8-0.8-0.9}$	0.011	$8.4^{+16.1}_{-7.2}$...
$B^+ \rightarrow f_2 K^+$	T,PA,P	$11.8^{+2.7+3.2+3.0}_{-2.4-2.8-2.7}$	0.34	$3.8^{+7.8}_{-3.0}$	$1.06^{+0.28}_{-0.29}$
$B^+ \rightarrow f' K^+$	P,PA	$3.8^{+0.4+0.9+1.0}_{-0.4-0.8-0.8}$	0.004	$4.0^{+7.4}_{-3.6}$	< 7.7
$B^+ \rightarrow K_2^{*+} \eta$	PA,P	$0.8^{+0.2+0.3+0.3}_{-0.2-0.2-0.3}$	0.031	$6.8^{+13.5}_{-8.7}$	9.1 ± 3.0
$B^+ \rightarrow K_2^{*+} \eta'$	PA,P	$12.7^{+3.7+4.5+4.0}_{-3.2-3.5-3.5}$	1.41	$12.1^{+20.7}_{-12.1}$	$28.0^{+5.3}_{-5.0}$
$B^0 \rightarrow K_2^{*+} \pi^-$	PA	$1.0^{+0.2+0.2+0.3}_{-0.2-0.2-0.2}$...	$3.3^{+8.5}_{-3.2}$	< 6.3
$B^0 \rightarrow K_2^{*0} \pi^0$	PA	$0.6^{+0.2+0.1+0.2}_{-0.1-0.1-0.1}$	0.084	$1.2^{+4.3}_{-1.3}$	< 4.0
$B^0 \rightarrow a_2^- K^+$	T,PA	$5.0^{+1.6+1.4+1.3}_{-1.4-1.1-1.0}$	0.58	$9.7^{+17.2}_{-8.1}$...
$B^0 \rightarrow a_2^0 K^0$	PA	$2.0^{+0.5+0.4+0.6}_{-0.5-0.4-0.5}$	0.005	$4.2^{+8.3}_{-3.5}$...
$B^0 \rightarrow f_2 K^0$	PA,P	$9.2^{+2.0+2.5+2.6}_{-1.8-2.1-2.2}$	0.005	$3.4^{+8.5}_{-3.1}$	$2.7^{+1.3}_{-1.2}$
$B^0 \rightarrow f'_2 K^0$	P,PA	$3.7^{+0.3+0.7+0.9}_{-0.4-0.8-0.9}$	0.00007	$3.8^{+7.3}_{-3.5}$...
$B^0 \rightarrow K_2^{*0} \eta$	PA,P	$1.0^{+0.2+0.3+0.3}_{-0.2-0.2-0.3}$	0.029	$6.6^{+13.5}_{-8.7}$	9.6 ± 2.1
$B^0 \rightarrow K_2^{*0} \eta'$	PA,P	$11.6^{+3.6+4.2+3.8}_{-2.9-3.1-3.1}$	1.30	$12.4^{+21.3}_{-12.4}$	$13.7^{+3.2}_{-3.1}$

topologies indicated through the symbols T (color-allowed tree), C (color-suppressed tree), P (penguin emission) and PA (penguin annihilation). Although we include also the W annihilation and W exchange diagram contributions, none of these channels has dominant contribution from these two topology. This is different from that of QCD factorization approach [25], where a large annihilation type contribution is introduced by hand to explain the large experimental data for the penguin annihilation channels. For the theoretical uncertainties in our calculation, we estimated three kinds of them: The first errors are caused by the uncertainties of the decay constants of tensor mesons. The second errors are from the decay constant ($f_B = (0.21 \pm 0.02)$ GeV) of B meson and the shape parameter ($\omega_B = (0.5 \pm 0.05)$ GeV) in the B meson wave function [5, 25]. The third errors are estimated from the unknown next-to-leading order QCD corrections and the power corrections, characterized by the choice of the $\Lambda_{QCD} = (0.25 \pm 0.05)$ GeV and the variations of the factorization scales, respectively. One can find that for most channels, the size of three kinds of theoretical uncertainties are comparative.

There are large theoretical uncertainties in any of the individual decay mode calculations. However, we can reduce the uncertainties by ratios of decay channels. For example, simple relations among some decay channels are derived in the limit of SU(3) flavor symmetry

$$\begin{aligned}
\mathcal{B}(B^0 \rightarrow K_2^{*0} \pi^0) &\sim \mathcal{B}(B^+ \rightarrow K_2^{*+} \pi^0) \sim \frac{1}{2} \mathcal{B}(B^0 \rightarrow K_2^{*+} \pi^-) \\
&\sim \frac{1}{2} \mathcal{B}(B^+ \rightarrow K_2^{*0} \pi^+), \\
\frac{\mathcal{B}(B^0 \rightarrow a_2^- K^+)}{\mathcal{B}(B^+ \rightarrow a_2^0 K^+)} &= \frac{\mathcal{B}(B^+ \rightarrow a_2^+ K^0)}{\mathcal{B}(B^0 \rightarrow a_2^0 K^0)} = 2.
\end{aligned} \tag{19}$$

One can find from table II that our results basically agree with the relation given above within the errors.

For decays involving one tensor meson and one heavy $D^{(*)}$ meson, we also give predictions with large branching ratios [27]. These B decays include the Cabibbo-Kobayashi-Maskawa- favored B decays through $b \rightarrow c$ transition, and the CKM-suppressed B decays through $b \rightarrow u$ transition. Since there are only tree operator contributions, no CP asymmetry appears in the standard model for these decays. Again, the factorizable diagrams with a tensor meson emitted vanish in the naive factorization. To deal with the large non-factorizable contribution and annihilation type contribution, one has to go beyond the naive factorization, to apply the perturbative QCD approach.

In charmed B decays, there is one more intermediate energy scale, the heavy D meson mass. As a result, another expansion series of m_D/m_B will appear. The factorization is approved at the leading of m_D/m_B expansion [8, 30]. It is also proved factorization in the soft collinear effective theory for this kind of decays [6]. Among those decays predicted in ref.[27], there are some pure annihilation type decays, such as $B^0 \rightarrow D_s^- K_2^{*+}$ and $B_s \rightarrow \bar{D} a_2$. There are currently no experimental measurements for these decays. For the first time, our perturbative QCD approach calculations give sizable predictions of branching ratios at the order of 10^{-5} and 10^{-6} , respectively [27], which may be measured soon in the experiments.

C. Two body hadronic decays of B_c meson

From a theoretical point of view [28], the non-leptonic decays of B_c meson are the most complicated decays due to its heavy-heavy nature and the participation of strong interaction, which complicate the extraction of parameters in SM, but they also provide great opportunities to study the perturbative and nonperturbative QCD, final state interactions and heavy quarkonium property, etc. It is well-known that B_c meson is a nonrelativistic heavy quarkonium system. Thus the two quarks in the B_c meson are both at rest and non-relativistic. Since the charm quark in the final state D meson is almost at collinear state, a hard gluon is needed to transfer large momentum to the spectator charm quark. In the leading order of $m_c/m_{B_c} \sim 0.2$ expansion, the factorization theorem is applicable to the B_c system similar to the situation of B meson [29]. Utilizing the k_T factorization instead of collinear factorization, the pQCD approach is free of endpoint singularity. Thus the diagrams including factorizable, nonfactorizable and annihilation type, are all calculable. For the charmed decays of B meson, it has been demonstrated to be applicable in the leading order of the m_D/m_B expansion [8, 30].

The two-body non-leptonic charmless decays $B_c \rightarrow PP, PV/VP, VV$ can occur through the weak annihilation diagrams only. The pQCD predictions [29] for the branching ratios vary in the range of 10^{-6} to 10^{-8} , basically agree with the predictions obtained by using the exact SU(3) flavor symmetry. The $B_c \rightarrow \bar{K}^{*0} K^+$ and other decays with a decay rate at 10^{-6} or larger could be measured at the LHC experiment. For $B_c \rightarrow PV/VP, VV$ decays, the branching ratios of $\Delta S = 0$ processes are basically larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.

Analogous to $B \rightarrow K\eta^{(\prime)}$ decays, we find $Br(B_c \rightarrow K^+\eta') \sim 10 \times Br(B_c \rightarrow K^+\eta)$. This large difference can be understood by the destructive and constructive interference between the η_q and η_s contribution to the $B_c \rightarrow K^+\eta$ and $B_c \rightarrow K^+\eta'$ decay. Because only tree operators are involved, the CP-violating asymmetries for these considered B_c decays are absent naturally. For $B_c \rightarrow VV$ decays, the longitudinal polarization fractions are around 95% to play the dominant role, except for $B_c \rightarrow \phi K^{*+}$ ($f_L \sim 86\%$).

For charmless decays involving scalar or axial vector final states, some calculations have already been done, such as $B_c \rightarrow AP$ decays, $B_c \rightarrow SP, SV$ decays, $B_c \rightarrow AV(VA)$ decays in the perturbative QCD approach [31] etc.

We calculate the CP averaged branching ratios and direct CP asymmetries for $B_c \rightarrow D_{(s)}P$ decays, together with results from the light-front quark model (LFQM) [32] and the relativistic constituent quark model (RCQM) [33], shown in table III, some of which with large direct CP asymmetry predictions. Generally, our predictions for the branching ratios in the tree-dominant B_c decays are in good agreements with that of RCQM model. But we have much larger branching ratios in the color-suppressed, annihilation diagram dominant B_c decays, due to the included non-factorizable diagrams and annihilation type diagrams contributions.

Other similar decay channels $B_c \rightarrow D_{(s)}V$ and the double charm decays of B_c meson are also calculated in ref.[34]. We find that the transverse polarization contributions in some channels, which mainly come from the non-factorizable

TABLE III: CP averaged branching ratios and direct CP asymmetries for $B_c \rightarrow D_{(s)}P$ decays, together with results from RCQM[33] and LFQM[32].

		$\mathcal{BR}(10^{-7})$			$A_{CP}^{dir}(\%)$	
channels	Class	This work	RCQM	LFQM	This work	RCQM
$B_c \rightarrow D^0 \pi^+$	T	$26.7^{+3.1+6.0+0.8}_{-3.5-5.6-0.6}$	22.9	4.3	$-41.2^{+4.5+11.1+0.8}_{-4.6-7.8-1.2}$	6.5
$B_c \rightarrow D^+ \pi^0$	C,A	$0.82^{+0.24+0.55+0.06}_{-0.16-0.41-0.01}$	2.1	0.067	$2.3^{+6.3+1.4+15.0}_{-3.0-0.8-18.8}$	-1.9
$B_c \rightarrow D^0 K^+$	A,P	$47.8^{+17.2+2.2+5.4}_{-9.1-1.7-3.6}$	44.5	0.35	$-34.8^{+4.9+7.4+1.8}_{-2.6-3.7-1.3}$	-4.6
$B_c \rightarrow D^+ K^0$	A,P	$46.9^{+15.6+0.3+7.4}_{-12.3-0.3-4.6}$	49.3	—	$2.3^{+0.4+0.9+0.0}_{-0.2-0.5-0.0}$	-0.8
$B_c \rightarrow D^+ \eta$	C,A	$0.92^{+0.15+0.21+0.03}_{-0.15-0.25-0.00}$	—	0.087	$40.8^{+0.0+18.4+15.6}_{-2.9-14.0-13.5}$	—
$B_c \rightarrow D^+ \eta'$	C,A	$0.91^{+0.12+0.16+0.06}_{-0.10-0.20-0.03}$	—	0.048	$-14.0^{+0.6+4.6+15.9}_{-1.5-5.2-11.9}$	—
$B_c \rightarrow D_s^+ \pi^0$	C,P	$0.41^{+0.04+0.01+0.02}_{-0.04-0.02-0.02}$	—	0.0067	$46.7^{+1.4+6.3+2.5}_{-1.4-11.8-2.8}$	—
$B_c \rightarrow D_s^+ \bar{K}^0$	A,P	$2.1^{+0.9+0.3+0.3}_{-0.6-0.3-0.2}$	1.9	—	$54.3^{+6.9+5.3+0.0}_{-7.2-8.0-0.3}$	13.3
$B_c \rightarrow D_s^+ \eta$	A,P	$17.3^{+1.7+0.5+3.3}_{-1.8-0.6-1.2}$	—	0.009	$2.8^{+0.0+0.4+1.1}_{-0.1-0.7-1.2}$	—
$B_c \rightarrow D_s^+ \eta'$	A,P	$51.0^{+4.9+0.4+6.7}_{-5.4-0.3-3.5}$	—	0.0048	$1.1^{+0.1+0.2+0.7}_{-0.0-0.2-0.6}$	—

emission diagrams or annihilation type diagrams, are large. The predicted branching ratios range from very small numbers of $\mathcal{O}(10^{-8})$ up to the largest branching fraction of $\mathcal{O}(10^{-5})$. The theoretical uncertainty study in the pQCD approach shows that our numerical results are reliable, which may be tested in the upcoming experimental measurements.

IV. SUMMARY

The current running of LHCb and other experiments measure more and more hadronic B_s and B_c decays, which require a precision theoretical study of these decays. We summarize the recent progress in theoretical study of two body non-leptonic B_s and B_c decays. Many of the next-to-leading order or even next-to-next-to-leading order α_s corrections have been performed in various approaches. Some of them are important in the phenomenological study and factorization study of hadronic B decays. We also emphasize the importance of the large power corrections in the heavy quark expansion, especially the large annihilation type contributions. These power corrections are essential in the study of direct CP asymmetry and polarization fraction study of two vector final states. When study the final states containing at least one scalar or tensor meson, these contributions may give the dominant contribution, especially in the QCD factorization approach. The reason is that the (axial-) vector current or (pseudo-) scalar density in the standard model can not produce a scalar or tensor meson from the vacuum. This makes the factorizable contribution to these decays negligible. Encouraged by the recent experimental confirmation of some B_s decays channels, a lot of study of the charmless and charmed B_s and B_c decays have been performed. Intensive study of decays involving a scalar, axial vector or tensor meson in the final states are also in the market. A future experimental measurement of these decays will give a test of the various factorization approaches.

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